Enrollment No: ______________________ Exam Seat No:______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Engineering Mathematics - IV

Subject Code : 4TE04EMT1 **Branch: B. Tech (Civil, Electrical, EC, Mech)**

Semester : 4 **Date : 01/10/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The Fourier cosine transform of $f(x)=5e^{-2x}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2+4}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{2}{\lambda^2+4}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{10}{\lambda^2-4}\right)$ (D) none of these

b) The Fourier sine transform of $f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}\left(\frac{1+\cos a\lambda}{\lambda}\right)$ (B) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda^2}\right)$ (C) $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos a\lambda}{\lambda}\right)$

(D) none of these

c) The image of circle $|z-1|=1$ in the complex plane, under the mapping $w=\frac{1}{z}$ is

(A) $|w-1|=1$ (B) $u^2+v^2=1$ (C) $v=\frac{1}{z}$ (D) $u=\frac{1}{z}$

d) If $w=\mathrm{f}(z)=\mathrm{u}(x,y)+i\mathrm{v}(x,y)$ is analytic then $f'(z)$ equal to

(A) $\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}$ (B) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$ (C) $\frac{\partial v}{\partial y}-i\frac{\partial v}{\partial x}$ (D) none of these

e) The magnitude of acceleration vector at $t=0$ on the curve $x=2\cos 3t, y=2\sin 3t$, $z=3t$ is

(A) 6 (B) 9 (C) 18 (D) 3

f) If $\vec{A}(t)=3t^2i+4tj+4t^3k$, $\int_{t=1}^{t=2}\vec{A}(t)\,dt$ equal to

(A) $15i+6j+7k$ (B) $7i+6j+5k$ (C) $7i+15j+6k$

(D) none of these

g) δ equal to

(A) $\frac{\Delta}{E^{\frac{1}{2}}}$ (B) $E^{\frac{1}{2}}+E^{\frac{-1}{2}}$ (C) $E^{\frac{1}{2}}-E^{\frac{-1}{2}}$ (D) none of these

h) E^{-1} equal to
(A) $1-\nabla$ (B) $1+\nabla$ (C) $1+\delta$ (D) $1-\delta$

i) The order of the difference equation $y_{n+3}-3y_{n+1}+2y_n=0$ is
(A) 1 (B) 2 (C) 3 (D) none of these

j) Putting $n=2$ in the Newton – Cote's quadrature formula following rule is obtained
(A) Simpson's $\frac{1}{3}$ rule (B) Trapezoidal rule (C) Simpson's $\frac{3}{8}$ rule
(D) none of these

k) The convergence in the Gauss – Seidel method is faster than Gauss – Jacobi method.
(A) True (B) False

l) The Gauss – Jordan method in which the set of equations are transformed into diagonal matrix form.
(A) True (B) False

m) The first approximation y_1 of the initial value problem $\frac{dy}{dx}=x^2+y^2$, $y(0)=0$ obtain by Picard's method is
(A) x^2 (B) $\frac{x^2}{2}$ (C) $\frac{x^3}{3}$ (D) none of these

n) Which of the following methods is the best for solving initial value problems:
(A) Taylor's series method (B) Euler's method
(C) Runge-Kutta method of 4^{th}order (D) Modified Euler's method

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Consider following tabular values **(5)**

x	50	100	150	200	250
y	618	724	805	906	1032

Using Newton's Backward difference interpolation formula determine $y(300)$.

b) Use Stirling's formula to find y_{28} given that **(5)**
$y_{20}=49225,\ y_{25}=48316,\ y_{30}=47236,\ y_{35}=45926$ and $y_{40}=44306$.

c) Find the finite Fourier sine transform of $f(x)=lx-x^2,\quad 0\le x\le l$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Solve the following system of equations by Gauss-Seidal method. **(5)**
$30x-2y+3z=75,\ 2x+2y+18z=30,\ x+17y-2z=48$

b) From the following table of values of x and y, find $\frac{dy}{dx}$ for $x=1.05$. **(5)**

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30

y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

c) Determine the analytic function whose imaginary part is $e^{x}\left(x\cos y - y\sin y\right)$ by Milne – Thompson method. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Use Euler's method to find $y(1.4)$ given that $\frac{dy}{dx} = xy^{1/2}$, $y(1)=1$. **(5)**

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 3/8 Rule using $h = \frac{1}{6}$. **(5)**

c) Solve the following system of equations by Gauss Elimination Method: **(4)**
$5x - 2y + 3z = 18,\ x + 7y - 3z = -22,\ 2x - y + 6z = 22$

Q-5 **Attempt all questions** **(14)**

a) Show that the function defined by the equation **(5)**

$$f(z) = \begin{cases} u(x,y) + iv(x,y), & \text{if } z \neq 0 \\ 0 \quad , & \text{if } z = 0 \end{cases}$$

where $u(x,y) = \frac{x^3 - y^3}{x^2 + y^2}$ and $v(x,y) = \frac{x^3 + y^3}{x^2 + y^2}$ is not analytic at $z = 0$ although Cauchy – Riemann equations are satisfied at that poiut.

b) Using Green's Theorem, evaluate $\int_C \left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy$ where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. **(5)**

c) The following table gives the values of x and y: **(4)**

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

Find the value of x corresponding to $y = 13.6$.

Q-6 **Attempt all questions** **(14)**

a) Prove that $\vec{F} = \left(y\cos z - \sin x\right)i + \left(x\sin z + 2yz\right)j + \left(xy\cos z + y^2\right)k$ is irrotational and find its scalar potential **(5)**

b) Under the transformation $w = \frac{1}{z}$ **(5)**

(a) Find the image of $\left|z - 2i\right| = 2$

(b) Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscates $\rho^2 = \cos 2\theta$.

c) Using Taylor's series method to solve $\frac{dy}{dx} = x^2 y - 1,\ y(0) = 1$. Also find $y(0.03)$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) If $f(z) = f(re^{i\theta}) = P(r,\theta) + iQ(r,\theta)$ is an analytic function, prove that both P and Q satisfy the Laplace equation in polar coordinates, namely **(5)**

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0.$$

b) If $\vec{F} = \left(2xy + z^3\right)\hat{i} + x^2\hat{j} + 3xz^3\hat{k}$, show that $\int_C \vec{F}\cdot d\vec{r}$ is independent of the path of integration. Hence evaluate the integral when C is any path **(5)**

joining A(1, – 2, 1) to B(3, 1, 4).

c) Use Trapezoidal rule to evaluate $\int_0^1 x^3\, dx$ considering five sub-intervals. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Use Runge-kutta second order method to find the approximate value of y(0.2) given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$ and $h = 0.1$. **(5)**

b) Using Fourier integral show that **(5)**

$$\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda = \begin{cases} \frac{\pi}{2} & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$$

c) Find the angle between the tangents to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at the points $t = 1$ and $t = 2$. **(4)**